Optimisation of Power Consumption in Wireless Network

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Introduction

- Problem Statement:Apply Queueing Theory in modelling the wireless network node and create a power consumption function along with user specified constraints and minimize it to obtain optimal system parameters.
- Why this work is relevant?
- Past research work

Background

- What is queue here?
- Significance of M/M/1
- N-policy M/M/1

State Diagram

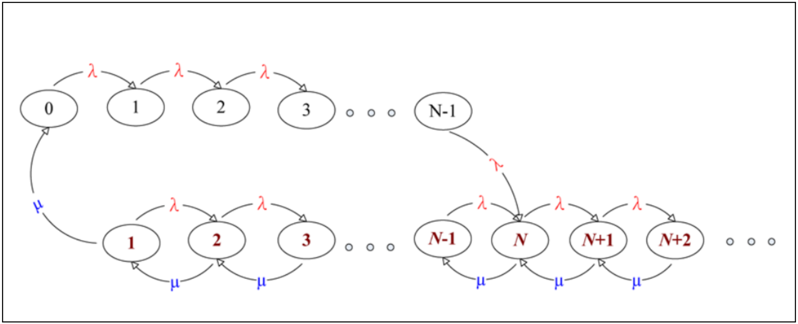


Figure: State diagram of the system at steady state

Equation at steady state

- System equations at equilibrium

$$\lambda P_0(0) = \mu P_1(1) \quad (1)$$

$$\lambda P_0(n) = \lambda P_0(n-1), 1 \leq n \leq N-1 \quad (2)$$

$$(\lambda + \mu) P_1(1) = \mu P_1(2) \quad (3)$$

$$(\lambda + \mu) P_1(n) = \lambda P_1(n-1) + \mu P_1(n+1), 2 \leq n \leq N-1 \quad (4)$$

$$(\lambda + \mu) P_1(N) = \lambda P_0(N-1) + \lambda P_1(N-1) + \mu P_1(N+1) \quad (5)$$

$$(\lambda + \mu) P_1(n) = \lambda P_1(n-1) + \mu P_1(n+1), n \geq N+1 \quad (6)$$

- Solution?

Moment generating function approach

- Moments at Idle, Busy and at any state.

$$G_I(z) = \sum_{n=0}^{N-1} z^n P_0(n), |z| \leq 1 \tag{7}$$

$$G_B(z) = \sum_{n=1}^{\infty} z^n P_1(n), |z| \leq 1 \tag{8}$$

$$G_L(z) = G_I(z) + G_B(z) \tag{9}$$

$$G_I(z) = P_0(0) \sum_{n=0}^{N-1} z^n = \frac{1 - z^N}{1 - z} P_0(0) \tag{10}$$

$$G_B(z) = \frac{\rho z(1 - z^N)}{\rho z^2 - (1 + \rho)z + 1} P_0(0) \tag{11}$$

Moment generating function approach

With $z = 1$ in $G_L(z)$, we get normalization condition and L'Hospital's rule

$$G_L(1) = G_I(1)+G_B(1) = \sum_{n=0}^{N-1} P_0(n)+\sum_{n=1}^{\infty} P_1(n) = \lim_{z\to 1} G_L(z) = \frac{N}{1-\rho}P_0(\tag{12}$$

Thus the probability of having no packet in system and system being in Idle state is given by $P_0(0) = \frac{1-\rho}{N}$.We denote P_I and P_B as probability of being in Idle and Busy state respectively. These probabilities can be derived easily using moment generating function.
$P_I = G_I(1) = \lim_{z\to 1} \frac{1-z^N}{1-z} P_0(0) = NP_0(0) = 1-\rho$ and
$P_B = G_B(1) = \lim_{z\to 1} \frac{\rho z(1-z^N)}{(1-\rho)(1-z)} P_0(0) = \rho$.

Performance measure

1. Expected number of packets in Idle state:

$$E(I) = \sum_{n=0}^{N-1} nP_0(n) = \frac{1-\rho}{N}\frac{N(N-1)}{2} = \frac{(N-1)(1-\rho)}{2} \tag{13}$$

2. Expected number of packets in Busy state:

$$E(B) = \sum_{n=0}^{\infty} nP_1(n) = G_B(1)' = \lim_{z\to 1} G_B(1)' = \frac{N\rho(1-\rho+\rho(1+\rho))}{2(1-\rho)} \tag{14}$$

3. Expected number of packets in N-policy M/M/1 queue:

$$E(L) = E(I) + E(B) = \frac{N-1}{2} + \frac{\rho}{1-\rho} \tag{15}$$

Performance measure

1. Idle Period:

$$E(T_I) = N/\lambda \tag{16}$$

2. Busy Period:

$$E(T_B) = \frac{N}{\mu(1-\rho)} \tag{17}$$

3. Busy Cycle:

$$E(T) = \frac{N}{\lambda(1-\rho)} \tag{18}$$

Cost function

- Introduction
- Assumptions
  1. Fixed energy consumption is incurred per busy cycle in switching between Idle and Busy state and vice verse.
  2. Energy is consumed for retaining the data packets present in the system.
  3. Constant amount of energy is consumed per Busy and Idle period in one Busy cycle.
  4. Server has the flexibility of increasing servicing rate but at the expense of increased energy consumption. Linear model is taken into consideration for simplicity.
- Function

$$F(N,\mu) = C_h E(L) + \frac{C_s}{E(T)} + C_{id}\frac{E(T_I)}{E(T)} + C_b\frac{E(T_B)}{E(T)} + P_s g(\mu) \quad (19)$$

Cost function construction

- Cost function

$$F(N,\mu) = C_h\left(\frac{N-1}{2} + \frac{\rho}{1-\rho}\right) + C_s\frac{\lambda(1-\rho)}{N} + C_{id}(1-\rho) + C_b\rho + P_s\mu \quad (20)$$

where $\rho = \frac{\lambda}{\mu}$.

- Any constraints?

Delay Function

- Delay function as a constraint

$$W_q = \frac{E(L')}{\lambda} \tag{21}$$

where $E(L')$ is expected number of packets waiting in the system.

$$E(L') = E(L) - \rho = \frac{N-1}{2} + \frac{\rho}{1-\rho} - \rho = \frac{N-1}{2} + \frac{\rho^2}{1-\rho} \tag{22}$$

Using equation (22) in (21), we obtain average delay per packet

$$W_q = \frac{N-1}{2\lambda} + \frac{\rho}{\mu - \lambda} \tag{23}$$

- Is it a QoS need?

Solving our Problem

- Problem Statement reduces to

$$\begin{aligned} \text{minimize} \quad & F(N, \mu) \\ \text{subject to} \quad & W_q \leq D \end{aligned} \tag{24}$$

  where D is the delay.
- Did I find a closed form solution using KKT?

No, Adopted numerical optimisation technique

- Iterative version - Basic Algorithm

$$x_{k+1} = x_k + \alpha_k p_k \tag{25}$$

- Algorithms completes in 2 steps

First part-without constraint

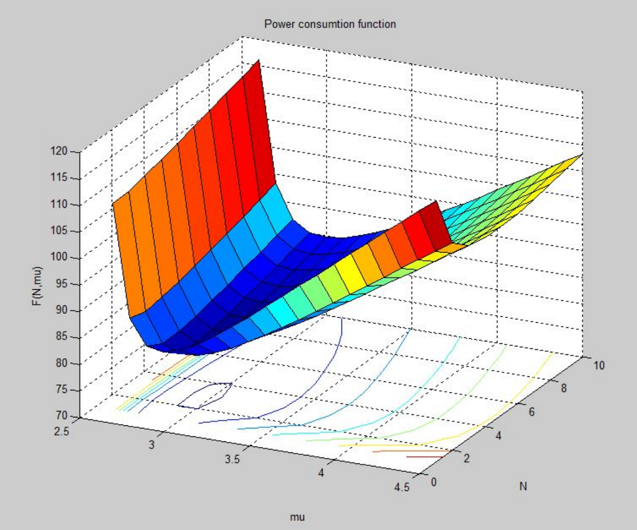
- Find p_k - Direction to move - Steepest decent
- Find distance α to move - Backtracking line search
- If solution is in feasible region, algorithm completed, else go to second part

Second part - with constraint

- Reduced two variable function $F(N, \mu)$ to one variable function $f(\mu)$ using equality constraint.
- Used same method with one dimension function to obtain optimal μ and corresponding N taking constraint into consideration.
- Return the solution

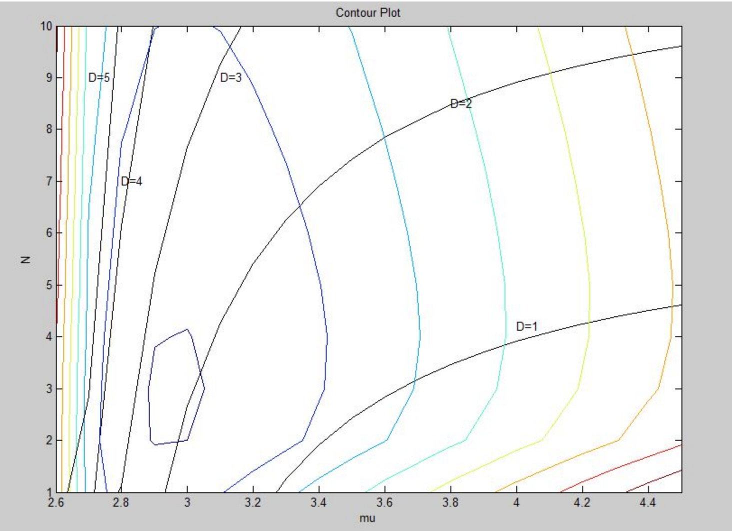
Mesh of cost function

- Surface plot of the cost function



Contour with different delay

- Contour plot with constraint D=1,2,3,4,5



Results

- Results with constants $C_h = 2,\ C_s = 20,\ C_{id} = 4,\ C_b = 5,\ P_s = 20$ and $\lambda = 2.5$.

	N^*	μ^*	$F(N^*, \mu^*)$
$D = 1$	1.7611	3.3740	86.0574
$D = 2$	2.5843	2.9958	79.6215
$D = 3$	2.7588	2.9484	79.4818
$D = 4$	2.7588	2.9484	79.4818
$D = 5$	2.7588	2.9484	79.4818

Table: Optimal parameters for different delay

Performance measurement metrics, PCIF

- Definition

$$PCIF = \frac{F(1, \mu_{min}) - F(N^*, \mu^*)}{F(1, \mu_{min})} * 100\% \qquad (26)$$

where $F(1, \mu_{min})$ represents the ordinary cost function with $N = 1$ and minimum average servicing rate. $F(N^*, \mu^*)$ is the cost value obtained after minimization.

- Results

	$F(N^*, \mu^*)$	$F(1, \mu_{min})$	PCIF
$D = 1$	86.0574	108.8846	20.9646%
$D = 2$	79.6215	108.8846	26.8753%
$D = 3$	79.4818	108.8846	27.0037%
$D = 4$	79.4818	108.8846	27.0037%
$D = 5$	79.4818	108.8846	27.0037%

Table: Percentage improvement over energy minimization

Some more analysis

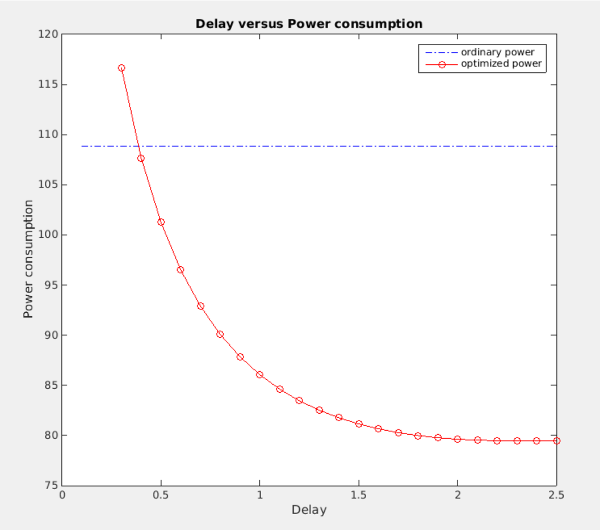


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Figure: Plot between Delay D and cost function $F(.)$

Some more analysis

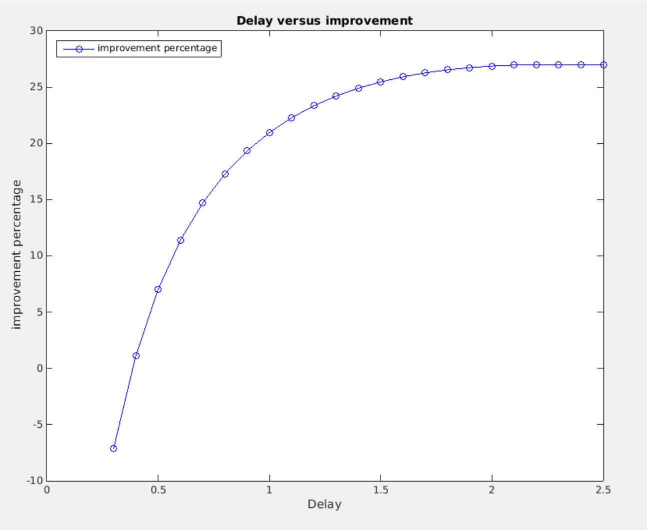

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Figure: Plot between Delay D and improvement % $PCIF$

Some more analysis

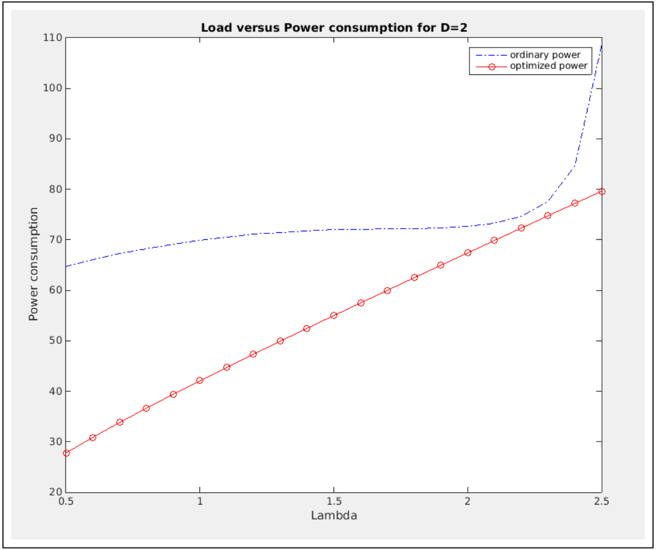

Figure: Plot between λ and cost function $F(.)$ at $D \leq 2$

Some more analysis

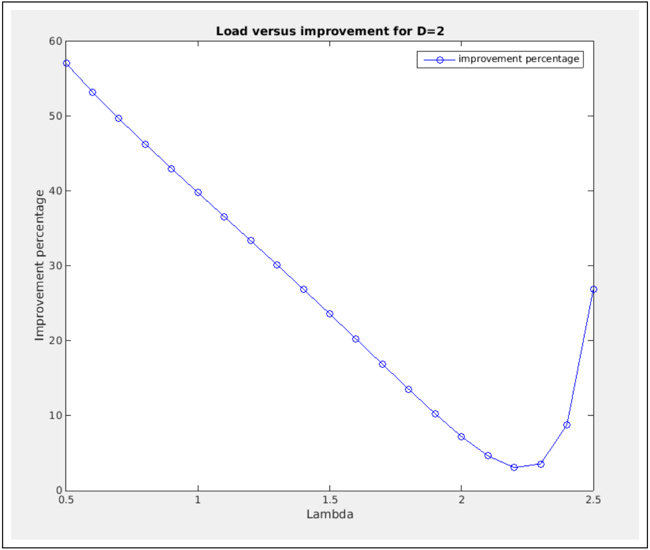

Figure: Plot between λ and improvement % $PCIF$ at $D \leq 2$

Future work

- Inclusion of a different yet important QoS constraint.
- Better numerical techniques.

Thank you